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ABSTRACT

A visual problem-solving technique applicable to several different classes of mechanics time-dependent problems is discussed. The computer is used to solve the equations of motion of various mechanical systems by one of several standard methods, and the solutions are displayed in time increments. A specific example is provided to illustrate this technique. (MLH)

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PICTORIAL SOLUTIONS IN ADVANCED MECHANICS

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Pictorial Solutions in Advanced Mechanics

Abstract

Due to the expansive amount of lecture theory workload and the lack of time available for peripheral problem-solving, innovative teaching techniques in advanced level mechanics courses frequently go unconsidered. Another difficulty is that at this level, few texts are standard and even fewer instructors follow these with exactness. The one aspect of advanced courses, in particular in advanced dynamics, which remains common to most classrooms is that the same problems are solved, or at least the governing equations are solved. Due to the complexity of such equations, however, a simplified case or two may be investigated briefly before new theory is begun.

What is described in this paper is a visual problem-solving technique applicable to several different classes of mechanics time-dependent problems. The computer is used to solve the equations of motion of various mechanical systems by one of several standard methods and the solutions are displayed in time increments. With this technique, important mechanical/physical parameters can be varied to give a parametric investigation capability to the classroom.

The problem demonstrated here is a two mass coupled system whose equations of motion form a set of

nonlinear, coupled differential equations of two degrees of freedom. The solution to this problem has been studied in detail, but mainly for cases where the nonlinear behavior is neglected, otherwise it generally defies classroom attention. First the initial conditions are prescribed and then the Runge-Kutta method is applied to extend the solution for each time step. The generalized coordinates are stored in a computer and finally a graphics routine is invoked so that at uniformly spaced time steps a picture is displayed or printed. If an interactive computer facility is available, the results may be viewed directly then electronically redrawn as time is augmented, otherwise motion picture film or videotape must be used to first copy each time frame and then to display the complete motion.

This technique can be useful in exploring the governing differential equations of many problems in the area of dynamics and vibrations. In particular the motion of other systems such as pendulums, gyroscopes, and rigid bodies is easily introduced into the classroom. All routines are written in Fortran and available upon request.

Purpose

The use of laboratory experiments to furnish a physical foundation for theory at the beginning level in Mechanics is well established. In fact it is customary to devote formal course time nearly equally between lecture and laboratory. Practical de-emphasis of laboratory however is eventually assumed by almost all students because course credit is applied to that aspect of the educational process very sparingly. The purpose of this paper therefore is to suggest a general method whereby continuing physical emphasis can be maintained in the advanced level mechanics curriculum without an inordinate penalty in instructional preparation time. This purpose is demonstrated by examples of several solutions following a presentation of the appropriate theory. It is hoped that the ease by which pictorial solutions may be applied to mechanics will generate an interest on the part of educators in using this technique and in encouraging the student to devote at least some attention to this method of investigating advanced mechanics.

Theory

The equations of motion for mechanical systems can be written generally as a set of differential equations in N generalized coordinates, q_i, as the dependent variables for systems with N degrees of freedom, $i = 1, 2, \ldots, N$, with the

one independent variable time. These systems are always 2nd order ordinary differential equations with variable coefficients, and non-linear in nature.[1] Using the Lagrangian formulation for deriving the equations of Motion; it is seen that the Lagrangian function, L, is at worst a function of q_i, $\dot{q}_i$, and t. Since single differentiation of the Lagrangian with respect to time gives the highest order derivatives of the q_i and $\dot{q}_i$'s ($\ddot{q}_i$), then eventually a system of equations results which is linear in the $\ddot{q}_i$'s, or

$$\text{since } \frac{d}{dt}\left[\frac{\partial L(q_i,\ \dot{q}_i, t)}{\partial \dot{q}_i}\right] - \frac{\partial L(q_i, \dot{q}_i, t)}{\partial q_i} = 0$$

then at worst

$$A_{ij}(q_j,\ \dot{q}_j, t)\ \ddot{q}_i = B_i(q_i,\ \dot{q}_i,\ t);$$

$$j = 1,\ \ldots,\ N.$$

(paired subscripts denote summation)

The A_{ij} coefficient matrix is not generally singular so that A_{ij}^{-1} exists leading to a set of N equations for the q_i's of the form:

$$\ddot{q}_i = f_i(q_j,\ \dot{q}_j,\ t);\ j = 1,\ \ldots,\ N.$$

When this form is established, an effective method must be utilized which by quadrature generates the q_i's as a function of time. A very general accurate and speedy process

is advantageous. Also, practical experience dictates the further condition that the integration step should be capable of being varied easily. These conditions fit the Runge-Kutta[2] Method attributes probably better than any other.

The advantage of this method technically over others for the programmer, a peripheral pursuit for most of us, is that prior solution values $q_i(t_n)$, $\dot{q}_i(t_n)$ are sufficient to determine the values $q_i(t_{n+1})$, $\dot{q}_i(t_{n+1})$. Thus values, for instance, at t_{n-1}, t_{n-2}, t_{n-3}, ..., are unnecessary and need not complicate the integration process or clutter machine memory, an important consideration in core-bound computers.

Although this method is normally stated for first-order differential equations, it may be extended simply [3] by restating the original problem slightly. For instance if there are N q_i's and N equations with (2N) initial conditions $q_i(t_o)$, $\dot{q}_i(t_o)$, then through the governing differential equations in q_i, the solution $q_i(t_1)$ and $\dot{q}_i(t_1)$ are determined from the solution at t_o with $t_1 = t_o + h$. In the general Runge-Kutta Method [a third order example is used], a change of variable is employed to generate 2N dependent variables as

$$\dot{q}_i = U_i$$

$$\dot{U}_i = f_i(q_j, U_j, t); \; j = 1, \ldots, N$$

with initial conditions $U_i(t_o)$, $q_i(t_o)$

The solution to this system at t_1 is:

let $k_i = hU_i(t_o)$ $\tilde{k}_i = hF_i(t_o, q_j(t_o), U_j(t_o))$; $j = 1, \ldots, N$

$\ell_i = h[U_i(t_o) + \frac{1}{2}\tilde{k}_i]$, $\tilde{\ell}_i = hF_i(t_o + \frac{1}{2}h, q_j(t_o) + \frac{1}{2}k_j,$

$U_j(t_o) + \frac{1}{2}\tilde{k}_j)$; $j = 1, \ldots, N$

$M_i = h[U_i(t_o) + \frac{3}{4}\tilde{\ell}_i]$, $\tilde{M}_i = hF_i(t_o + \frac{3}{4}h, q_j(t_o) + \frac{3}{4}\ell_j,$

$U_j(t_o) + \frac{3}{4}\tilde{\ell}_j)$; $j = 1, \ldots, N$

then $q_i(t_1) = q_i(t_o) + \frac{1}{9}(2k_i + 3\ell_i + 4m_i)$

$U_i(t_1) = U_i(t_o) + \frac{1}{9}(2\tilde{k}_i + 3\tilde{\ell}_i + 4\tilde{m}_i)$

at this point $h = t_2 - t_1$, can be changed, and the process repeated.

Procedure

This method is very useful if a routine exists which can transform the q_i's into a graphical solution at selected time steps. These time steps are chosen so that they match framing speeds of conventional cameras, where typically $h \approx .05$ sec., or CRT or video equipment. In some complex cases, the h used for the video work may be a multiple of the numerical solution time step.

The computer coding subroutines exist at this writing for picturing circles, or spheres, and linkages or bars. These are adequate to generate most systems commonly used in classroom examples. The main computer program solves the equations of motion for the number of time steps until another print step is reached then the q_i's are used to evaluate the center point coordinates and radius of any circles to be printed as well as the two end pin connections and width of each linkage. Next, the main computer program calls these generation subroutines which clear the previous picture from storage and form the new image. At this writing printer output has been utilized on a 10 inch by 10 inch grid with a caption, but the method is more efficient on CRT displays or on plasma screens where available.

The computer program is available as written in FORTRAN IV and requires only approximately 100 K bytes of core. Suggested types of mechanical systems which can be easily depicted are:

Pendulums

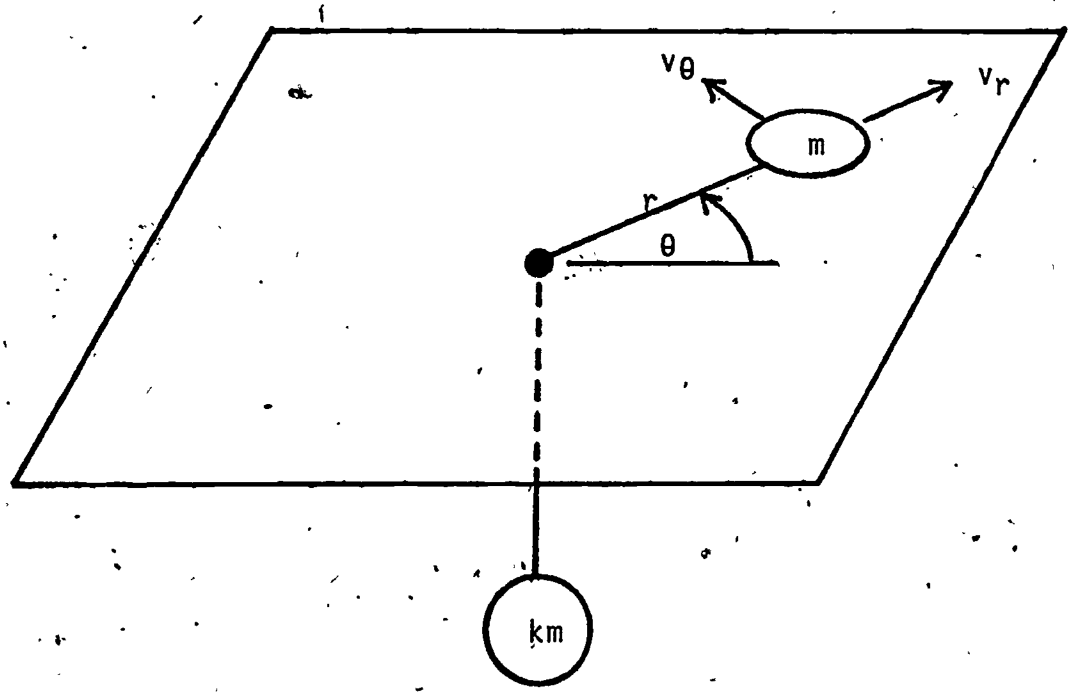
Tops

N-body Central Force Problems

Spring Mass Systems.

Sample Problem

A classical problem involving two masses coupled by a string is considered. One mass, m, is allowed to move on a frictionless table while a second mass, km, is attached through a hole in the table by means of an inextensible cord.



Two Mass System

The equations of motion [4] are given by

$(1 + k)\ddot{r} - r\dot{\theta}^2 = -kg$ and $2\dot{r}\dot{\theta} + r\ddot{\theta} = 0$ and subject to $\theta(t_o)$, $\dot{\theta}(t_o)$, $r(t_o)$, $\dot{r}(t_o)$.

Letting the Runge-Kutta variables be

$q_1 = \theta$, $q_2 = r$, $U_1 = \dot{\theta}$, $U_2 = \dot{r}$, and $N = 2$;

The algorithm becomes:

$$k_1 = h.U_1(t_o)$$

$$k_2 = h\ U_2(t_o)$$

$$\ell_1 = h[U_1(t_o) + \tfrac{1}{2}\tilde{k}_1]$$

$$\ell_2 = h[U_2(t_o) + \tfrac{1}{2}\tilde{k}_2]$$

$$m_1 = h[U_1(t_o) + \tfrac{3}{4}\tilde{\ell}_1]$$

$$m_2 = h[U_2(t_o) + \tfrac{3}{4}\tilde{\ell}_2]$$

$$\tilde{k}_1 = -2hU_1(t_o)\ U_2(t_o)/q_2(t_o)$$

$$\tilde{k}_2 = h(q_2(t_o)\ U_1^2\ (t_o) - kg)/(1 + k)$$

$$\tilde{\ell}_1 = -2h(U_1(t_o) + \tfrac{1}{2}\tilde{k}_1)(U_2(t_o) + \tfrac{1}{2}\tilde{k}_2)/(q_2(t_o) + \tfrac{1}{2}k_2)$$

$$\tilde{\ell}_2 = h((q_2(t_o) + \tfrac{1}{2}k_2)(U_1(t_o) + \tfrac{1}{2}\tilde{k}_1)^2 - kg))/(1 + k)$$

$$\tilde{m}_1 = -2h((U_1(t_o) + \tfrac{3}{4}\ell_1)(U_2(t_o) + \tfrac{3}{4}\tilde{\ell}_2))/(q_2(t_o) + \tfrac{3}{4}\ell_2)$$

$$\tilde{m}_2 = h((q_2(t_o) + \tfrac{3}{4}\ell_2)(U\ (t_o) + \tfrac{3}{4}\tilde{\ell}_1)^2 - kg)/(1 + k)$$

$$q_1(t_1) = q_1(to) + \tfrac{1}{9}(2k_1 + 3\ell_1 + 4m_1)$$

$$q_1(t_1) = q_2(t_o) + \tfrac{1}{9}(2k_2 + 3\ell_2 + 4m_2)$$

$$U_1(t_1) = U_1(t_o) + \tfrac{1}{9}(2\tilde{k}_1 + 3\tilde{\ell}_1 + 4\tilde{m}_1)$$

$$U_2(t_1) = U_2(t_o) + \tfrac{1}{9}(2\tilde{k}_2 + 3\tilde{\ell}_2 + 4\tilde{m}_2)$$

Initial conditions used for the computer generated pictures were:

r_0 = 1.0 feet θ_0 = 0.0 radians

$\dot{r}_0$ = 10.0 feet/sec. $\dot{\theta}_0$ = 3.0 radians/sec.

k = 4.0   g = 32.2 feet/sec.^{2}

to (time at start of motion) = 0.0 sec.

t_{max} (time at end of motion) = 10.0 sec.

h = .001 seconds

Print step at every .05 seconds.

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